Electric energy conversion to mechanical work

Every electric machine has three independent circuits: primary electric circuit E_1, magnetic circuit M and secondary electric circuit E_2. There is in simple scheme on Fig. 1 illustrated coupling between electric circuits E_1 and E_2 through the magnetic one M.

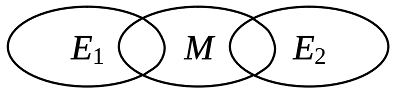


Fig. 1 Three basic parts of electric machine

Principle of electro-mechanic conversion in asynchronous motor

Asynchronous motor operation in function of motor is simple and it is possible to present it as follows (see Fig. 1). Three-phase stator winding is connected to three-phase supply system (E_1). Through the coils flow currents with phase turning 120 °, and they initiate in magnetic circuit (M) magnetic field, which turns so-called synchronous revolutions n_1. Induction lines of magnetic field cut conductors of both electric circuits (E_1 and E_2) and so they induce voltages in them. Induced voltage in stator winding affect against induced voltage of supply system, and by that fact it limits stator's currents to values, for which is motor constructed.

Induced voltage in rotor's winding (E_2) initiates in it current and because it is in influence of magnetic field, it affects the mechanical force on winding as on each current-conductor in magnetic field. Conductors of rotor's winding then begin to deviate in direction of magnetic field rotation (Lenz's law).

After sequential increasing of revolutions the relative rotor's conductors motion is decreasing compared to stator field. Then there is decelerated a time change of magnetic flow coupled by conductors, there is decreasing induced voltage and also current, what the result from that fact is also decreasing of mechanical force affected to conductors. There occurs to consolidation of rotor revolutions below synchronous value. Therefore this motor is named as asynchronous or inductive. Inductive for that fact, that its influencing is based on voltage inducing in rotor from magnetic field initiated by stator. From presented description the brief result can be expressed as follows: electric energy delivered from the power supply system to stator is by the created rotated magnetic field transferred on rotor and from that fact it is transferred in mechanical form to shaft of driven device. Electric machine works as the asynchronous motor (with rotations smaller than synchronous), on rotor shaft is generated motive rotating moment.

Rotating magnetic field, revolutions of asynchronous motor

Demonstrative electromagnetic scheme of three-phase asynchronous machine is on Fig. 2.

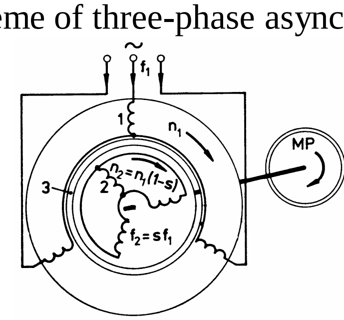


Fig. 2 Scheme of three-phase asynchronous machine

Magnetic circuit that consists from laminations is composed from two co-axial cylinders. Internal cylinder – rotor is fixed on shaft and is rotating in bearing inside of external cylinder – stator.

Stator's winding 1 is uniformly spaced in stator's drains with phase shifting one to another about 120 °. Rotor's winding 2 is either three-phase or multi-phase and in the simplest case it is short-circuited. By reason of good magnetic coupling between stator and rotor the air gap 3 must be as small as it is possible. As we indicate it before, if we connect stator's winding to the three-phase current source, this generates the magnetic field. In given space configuration of winding coils and properties of three-phase current source the generated magnetic field is moving even so, as we rotate the permanent magnet around the axis. Because of that it is named rotating magnetic field. Rotating field induces in rotor's winding the electromotive voltage that initiates currents in closed winding. These ones, together with rotating magnetic field generate electromagnetic forces, which rotate rotor in direction of field rotating.

If magnetic field activated by alternate current around the stator circumference has one north and one south pole (number of machine poles is $2p = 2$, therefore number of pole pairs is $p = 1$), then time of one current oscillation corresponds to field rotating in 360 °. If number of synchronous field revolutions per 1 minute is n_1, corresponding frequency is

$$f_1 = \frac{n_1}{60} \qquad [\text{Hz; min}^{-1}] \qquad (1)$$

what is frequency of stator current. Then, synchronous revolutions of two-pole machine in main frequency 50 Hz are

$$n_1 = 60 \cdot f_1 = 3000 \text{ ot.min}^{-1} \qquad (2)$$

Synchronous revolutions with multi-pole pairs are

$$n_1 = \frac{60 \cdot f_1}{p} \qquad [\text{min}^{-1}] \qquad (3)$$

Some specific values of revolutions n_1 in main frequency $f_1 = 50$ Hz and different numbers of machine's pole are presented in next table

Number of poles $2p$	**Number of pole pairs p**	**Synchronous revolutions**	
		per second (s^{-1})	**per minute (min^{-1})**
2	1	50	3000
4	2	25	1500
6	3	16,67	1000
8	4	12,5	750
10	5	10	600
12	6	8,33	500

If in steady state the rotor revolution compared to stator revolution are denoted n_2 [s^{-1}], then difference

$$n_s = n_1 - n_2 \qquad [\text{s}^{-1}] \qquad (4)$$

defines the relative revolutions of rotor compared to rotating field of stator. They are named ***slip speed***. Slip speed is corresponding to frequency of induced voltage in rotor's winding

$$f_2 = p \cdot n_s \qquad [\text{Hz; s}^{-1}] \qquad (5)$$

Slip speed also defines so-called ***asynchronous machine slip*** as a proportion of slip speed to synchronous revolutions

$$s = \frac{n_s}{n_1} = \frac{n_1 - n_2}{n_1} \tag{6}$$

where real rotor revolutions per 1 minute using by expression (3) are

$$n_2 = n_1 \cdot (1 - s) = \frac{60 \cdot f_1}{p} \cdot (1 - s) \qquad [\text{min}^{-1}] \tag{7}$$

Because the asynchronous machines work with slip $s << 1$, from expression (7) results, those revolutions of rotor are close to synchronous ones. Slip values are approximately 5 %, in smallest motors is around 10 %, in biggest motors is about 1 %.

If rotor's winding is connected, also in it flows the three-phase current that analogically generates rotating magnetic field. The field, regarding to rotor is rotating with slip speed revolutions

$$n_s = \frac{60 \cdot f_2}{p} \qquad [\text{min}^{-1}] \tag{8}$$

and regarding to stator with revolutions

$$n_1 = n_s + n_2 \qquad [\text{min}^{-1}] \tag{9}$$

Synchronous revolutions are therefore the sum of slip speed revolutions and rotor's revolutions, eventually in other words: both of rotating fields is added into result, mutual magnetic field, which is rotating with synchronous revolutions n_1.

If the last equation is expressed by frequencies, then

$$\frac{60 \cdot f_1}{p} = \frac{60 \cdot f_2}{p} + n_2 \tag{10}$$

where rotor frequency is

$$f_2 = f_1 - p \cdot \frac{n_2}{60} \qquad [\text{Hz; min}^{-1}] \tag{11}$$

Because the rotor revolutions n_2 can be positive or negative (positive, if rotor is rotating in direction of field; negative, if rotor is rotating against the direction of field), rotor frequency can be higher or smaller as the supply system frequency. Asynchronous machine can therefore work as frequency converter.

After all, from equation (6) by using (3) and (8) results, that

$$s = \frac{f_2}{f_1} \qquad \text{eventually} \qquad f_2 = s \cdot f_1 \tag{12}$$

Dependency of frequency f_2 on slip and revolutions is illustrated in Fig. 3

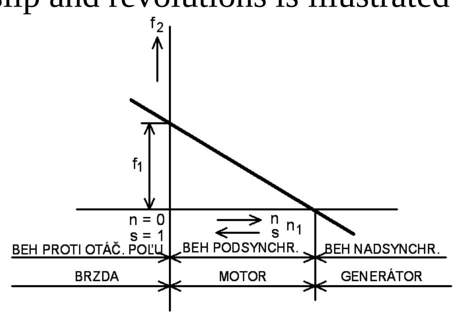


Fig. 3 Dependency of frequency on slip and revolutions

Fig. 3 illustrates three basic operation states of asynchronous machine, i.e. in function like a motor, generator and brake.

Energetic balance and efficiency of asynchronous motor

Energetic balance of asynchronous motor formulated by power is illustrated on Fig. 4

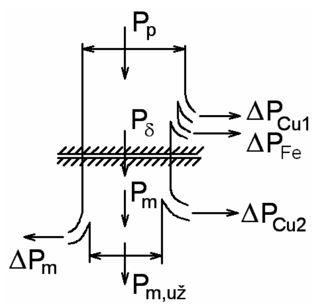


Fig. 4 Electric power distribution of asynchronous motor

Considering the labeling on Fig. 4, the equation of energetic (power) balance is

$$P_{\mathrm{p}} = \Delta P_{\mathrm{Cu1}} + \Delta P_{\mathrm{Fe}} + \Delta P_{\mathrm{Cu2}} + \Delta P_m + P_{\mathrm{m,už}} \qquad [\mathrm{W}] \tag{13}$$

Particular components represent:

- P_{p} is electric input motor power, because it is appliance that symmetrically load the three-phase supply network, it is expressed by well-known expression
  $$P_{\mathrm{p}} = 3 \cdot \boldsymbol{U}_1 \cdot \boldsymbol{I}_1 \cdot \cos j_1 \tag{14}$$
  for phase values of stator voltage and current $\boldsymbol{U}_1$, $\boldsymbol{I}_1$ and power factor $\cos j_1$.
- Stator current in its winding with resistance R_1 generates electric losses
  $$\Delta P_{\mathrm{Cu1}} = 3 \cdot R_1 \cdot I_1^2 \tag{15}$$
- ΔP_{Fe} represents losses in stator iron, that consist of hysteresis losses and losses by eddy currents
  $$\Delta P_{\mathrm{Fe}} = \Delta P_{\mathrm{h}} + \Delta P_{\mathrm{v}} \tag{16}$$

Through the rotating magnetic field on rotor there is transferred power

$$P_d = P_{\mathrm{p}} - \Delta P_{\mathrm{Cu1}} - \Delta P_{\mathrm{Fe}} \tag{17}$$

It is named power in air gap, which if we neglect the losses in iron (they are small) can be moreover divided into

- ΔP_{Cu2}, what represents the electric losses in rotor winding with resistance R_2 and for example the power consumed by trigger, eventually by speed regulator (resistance R_{sp}). Hence
  $$\Delta P_{\mathrm{Cu2}} = 3 \cdot \left(R_2 + R_{\mathrm{sp}}\right) \cdot I_2^2 \tag{18}$$
- and into mechanical power
  $$P_{\mathrm{m}} = P_d - \Delta P_{\mathrm{Cu2}} \tag{19}$$

If we subtract from it mechanical losses ΔP_{m} (bearing friction, ventilation, ...) we get total, mechanical useful power on motor shaft

$$P_{\mathrm{m,už}} = P_{\mathrm{m}} - \Delta P_{\mathrm{m}} \tag{20}$$

From the indicated balance it is possible simply determine efficiency of asynchronous motor

$$h = \frac{P_{\text{m,už}}}{P_{\text{p}}} = \frac{P_{\text{p}} - \left(\Delta P_{\text{Cu1}} + \Delta P_{\text{Fe}} + \Delta P_{\text{Cu2}} + \Delta P_{\text{m}}\right)}{P_{\text{p}}} \tag{21}$$

It is necessary to emphasize, that the equation (21) expresses total, therefore energetic efficiency of electromotor. If we would like to express electric efficiency, then in equation it is necessary to skip mechanical losses ΔP_{m}. During normal motor state are these losses essentially smaller as the sum $\Delta P_{\text{Cu1}} + \Delta P_{\text{Fe}} + \Delta P_{\text{Cu2}}$, so in the final result we can consider the energetic efficiency as the electric efficiency in electromechanical conversion. Efficiency values of asynchronous motors are in the range of 75 to 93 %.

Torque of asynchronous motor

There is a universal relation among the power, revolution and torque M

$$P = w \cdot M = 2 \cdot p \cdot n \cdot M \tag{22}$$

where n are revolutions per 1 second. For expression of asynchronous machine torque is determining the power in air gap, which is transferred by magnetic field with revolutions n_1, therefore in agreement with (22), is then

$$P_d = 2 \cdot p \cdot n_1 \cdot M \qquad [\text{W; s}^{-1}\text{, N.m}] \tag{23}$$

Electric rotor power must be naturally transferred by the same torque, but by the other revolutions. These are the field revolutions compared to rotor, i.e. slip revolutions n_s. Therefore electric power in rotor (18) consumed in form of losses is also function of slip revolutions:

$$\Delta P_{Cu2} = 2 \cdot p \cdot n_s \cdot M = 2 \cdot p \cdot s \cdot n_1 \cdot M = s \cdot P_d \tag{24}$$

The rest of electric rotor power is total mechanical power on motor shaft (19), with revolutions n_2:

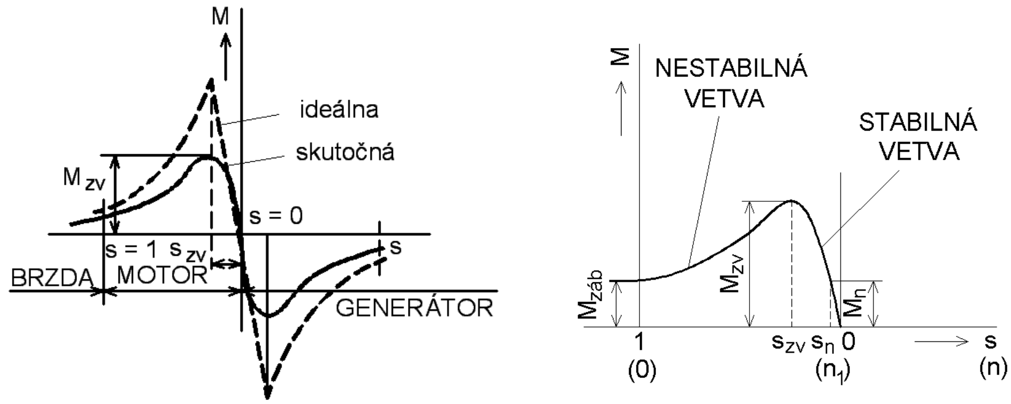
$$P_{\text{m}} = 2 \cdot p \cdot n_2 \cdot M = 2 \cdot p \cdot n_1 \cdot (1 - s) \cdot M = (1 - s) \cdot P_d \tag{25}$$

From the last two equations and from equation (19) we get

$$P_d = P_{\text{m}} + \Delta P_{\text{Cu2}} = (1 - s) \cdot P_d + s \cdot P_d \tag{26}$$

what means, that the power distribution in air gap into mechanical and electric (for coverage of losses ΔP_{Cu2}) depends on slip. Slip of asynchronous machine must be because of that as small as it is possible.

Calculation of torque magnitude of asynchronous machine is performed according to expression (23), in which the power in air gap P_d is calculated from the electric values of machine substitution diagram. Their mutual relation expresses the torque characteristics. There is illustrated the objected characteristics of asynchronous machine On Fig. 5.



a) b)

Fig 5 Torque dependency on slip of: a) asynchronous machine b) asynchronous motor

Similarly as from the progress of function f_2 = f(s) (Fig. 3) also from Fig. 5 result 3 operation states of asynchronous machine. In the slip range $0 < s < 1$ works in sub-synchronous state, therefore in function of motor. Motor cannot reach synchronous revolutions ($s = 0$), because in this state the relative rotor motion compared to the field would be equal to zero, in its conductors wouldn't be induced any voltage, through the conductors wouldn't flow any current, so there wouldn't influence any mechanic force, that initiate torque on the shaft. In function of generator the asynchronous machine works in that case, if on the rotor is transported mechanical energy through the driving machine, by which the rotor can rotate with over-synchronous revolutions ($n_2 > n_1$; $s < 0$). Mechanical rotor energy is through the magnetic field (by electromagnetic induction) changed in stator winding to electric form, from where it is transferred to supply system. In comparison to motor, in generator there is occurred the change of energy flow direction, what requires the change of meaning of relative conductor motion of rotor in comparison to rotating field. This fact is achieving by over-synchronous revolutions of asynchronous machine (by slip changing), which in this state develops the generator (brake) torque. This two-direction change of these conversion energy forms is denoted as the ***principle of electric machine reversibility***.

At last, the third basic operation state of asynchronous machine is in function of brake. It happens in that time, when by the load, which is necessary to brake there are forced revolutions to machine against the rotating field meaning. Then the machine produces brake torque. In this state the slip $s > 1$ and the rotor revolution n_2 are "negative" (opposite direction in comparison to n_1).

Characteristic points of torque machine characteristic for motor state are illustrated on Fig. 5b. In the state of rotor silent ($s = 1$) the machine generates the starting torque M_{zab}, that size is 1 to 2,5-multiple of nominal torque M_n. By this torque the motor is starting, the revolutions n_2 are increasing, slip is decreasing, and torque is rising up to breakdown torque M_{zv}, what is the maximal motor's torque. This part of the torque characteristics is unstable and motor operation is unwanted. It is because, that by the load increasing on the motor shaft the revolutions n_2 are decreasing, the slip is increasing, rotating torque is decreasing until the motor can stop. Stable part of the characteristics is slip region in interval $s \in \langle s_{zv}; 0 \rangle$, in which is the value of nominal torque M_n. In the range of this characteristics line the rotating torque is increasing and motor doesn't stop when slip is increasing (by decreasing of n_2 caused by increasing load on the shaft). The proportion of the breakdown torque and nominal torque is named as torque overload capacity, which size is about 2.

Example 1

Asynchronous three-phase 4-poles ($2p = 4$) motor is connected to the supply network with frequency $f_1 = 50$ Hz. Determine the synchronous revolutions of rotating stator magnetic field n_1 (rpm), rotor slip s, frequency of induced rotor current f_2 and mechanic angle speed of rotor w, if the rotor revolutions $n = 1440$ ot.min^{-1}.

Solution:

$$n_1 = \frac{60 \cdot f_1}{p} = \frac{60 \cdot 50}{2} = 1500 \text{ rpm}$$

$$s = \frac{n_1 - n}{n_1} = \frac{1500 - 1440}{1500} = 0{,}04 \Rightarrow 4\,\%$$

$$w = \frac{2 \cdot p \cdot n}{60} = \frac{2 \cdot p \cdot 1440}{60} = 150{,}8 \text{ s}^{-1}$$

$$f_2 = f_1 \cdot s = 50 \cdot 0{,}04 = 2 \text{ Hz}$$

Example 2

Asynchronous three-phase motor for voltage $U_s = 400$ V, $f_1 = 50$ Hz, has stator's winding connected to wye (Y), the power $P = 7{,}5$ kW, nominal revolutions $n = 1455$ rpm, efficiency with respecting all the losses $h = 86$ % and power factor $\cos j = 0{,}88$. Determine the number of poles ($2p$) of motor, rotor slip s, input power P_p, current I_1 and rotating torque M.

Solution:

The nearest higher synchronous revolutions of rotating stator magnetic field n_1, according to rotor revolutions $n = 1455$ rpm, must satisfy to equation

$$n_1 = \frac{60 \cdot f_1}{p} = \frac{60 \cdot 50}{2} = 1500 \text{ rpm} \quad \Rightarrow \quad \text{motor has 4-poles } 2p = 4$$

$$s = \frac{n_1 - n}{n_1} = \frac{1500 - 1455}{1500} = 0{,}03 \Rightarrow 3\,\%$$

$$P_p = \frac{P}{h} = \frac{7500}{0{,}86} = 8721 \text{ W}$$

When stator's winding is connected to wye (Y), we can determine the current by the expression:

$$I_1 = \frac{P_p}{\sqrt{3} \cdot U_s \cdot \cos j} = \frac{8721}{\sqrt{3} \cdot 400 \cdot 0{,}88} = 14{,}3 \text{ A}$$

$$M = \frac{P}{w} = \frac{P}{\frac{2 \cdot p \cdot n}{60}} = \frac{7500}{\frac{2 \cdot p \cdot 1455}{60}} = 49{,}22 \text{ N} \cdot \text{m}$$

Example 3

Asynchronous three-phase, 2-poles motor, is powered from frequency converter $f_1 = 60$ Hz. Determine the motor power and Joule's losses in rotor winding ΔP_{j2}, when the input power is $P_p = 2400$ kW and revolutions $n = 3456$ rpm.

Solution:

$$s = \frac{n_1 - n}{n_1} = \frac{3600 - 3456}{3600} = 0{,}04 \Rightarrow 4\,\%$$

$$n_1 = \frac{60 \cdot f_1}{p} = \frac{60 \cdot 60}{1} = 3600 \text{ rpm}$$

$$P = (1 - s) \cdot P_\mathrm{p} = (1 - 0{,}04) \cdot 2400 = 2304 \text{ W}$$

$$\Delta P_{\mathrm{j2}} = s \cdot P_\mathrm{p} = 0{,}04 \cdot 2400 = 96 \text{ W}$$

Example 4

Asynchronous three-phase 8-poles motor, which is powered from frequency converter f_1 = 40 Hz has rotation n = 570 rpm. Determine the synchronous revolutions of rotating stator magnetic field n_1, slip s, power P, Joule's losses in rotor's winding ΔP_{j2} and efficiency h, when motor shows the rotating torque M = 60 N·m.

Solution:

$$n_1 = \frac{60 \cdot f_1}{p} = \frac{60 \cdot 40}{4} = 600 \text{ rpm}$$

$$s = \frac{n_1 - n}{n_1} = \frac{600 - 570}{600} = 0{,}05 \Rightarrow 5\,\%$$

$$P = M \cdot w = 60 \cdot \frac{p}{30} \cdot 570 = 3581{,}4 \text{ W}$$

$$P = (1 - s) \cdot P_\mathrm{p} \qquad \Rightarrow \qquad P_\mathrm{p} = \frac{P}{1 - s} = \frac{3581{,}4}{1 - 0{,}05} = 3769{,}9 \text{ W}$$

$$\Delta P_{\mathrm{j2}} = s \cdot P_\mathrm{p} = 0{,}05 \cdot 3769{,}9 = 188{,}5 \text{ W}$$

$$h = \frac{P}{P_\mathrm{p}} = 1 - s = 1 - 0{,}05 = 0{,}95$$

Example 5

Asynchronous three-phase, 4-poles motor has in considered load the input power P_p = 565 W, frequency of rotor current f_2 = 1,5 Hz and slip s = 0,05. Determine the voltage frequency of power source f_1, revolutions n, losses ΔP_{j2}, power P and rotating motor torque M.

Solution:

$$f_2 = s \cdot f_1 \qquad \Rightarrow \qquad f_1 = \frac{f_2}{s} = \frac{1{,}5}{0{,}05} = 30 \text{ Hz}$$

$$n_1 = \frac{60 \cdot f_1}{p} = \frac{60 \cdot 30}{2} = 900 \text{ rpm}$$

$$s = \frac{n_1 - n}{n_1} \qquad \Rightarrow \qquad n = n_1 \cdot (1 - s) = 900 \cdot (1 - 0{,}05) = 855 \text{ rpm}$$

$$\Delta P_{\mathrm{j2}} = s \cdot P_\mathrm{p} = 0{,}05 \cdot 565 = 28{,}25 \text{ W}$$

$$P = (1 - s) \cdot P_\mathrm{p} = (1 - 0{,}05) \cdot 565 = 536{,}75 \text{ W}$$

$$M = \frac{P}{w} = \frac{P}{\frac{2 \cdot p \cdot n}{60}} = \frac{536{,}75}{\frac{2 \cdot p \cdot 855}{60}} \cong 6 \text{ N} \cdot \text{m}$$