Recapitulation:

Heat is a form of energy, which can be characterized as an interaction between hot and cold body or as the energy of vibrating molecules.

Heat transfer is realized by:

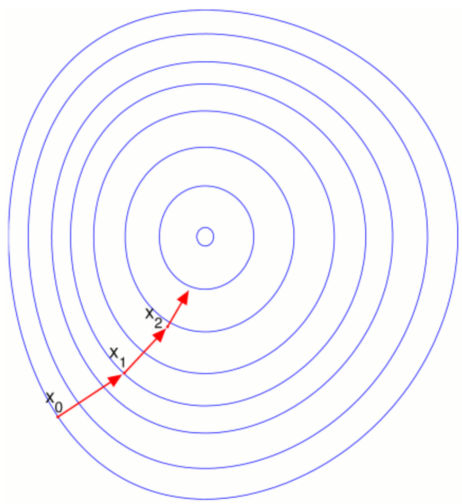
- ***conduction***
- ***convection***
- ***radiation***

Generally, heat transfer is realized by combination of two or more method above.

Why study heat transfer?

In many engineering situations we are interested in either *enhancing* heat transfer (eg in heat-exchangers), or in *inhibiting* heat transfer (e.g. in loft insulation). For solving of there processes we use heat transfer processes stated above.

The gradient of a scalar field is a vector field which points in the direction of the greatest rate of increase of the scalar field, and whose magnitude is the greatest rate of change.



By definition, the gradient is a vector field whose components are the partial derivatives of J. That is

$$\operatorname{grad} J = \lim_{\Delta n \to 0} \frac{\Delta J}{\Delta n} \cdot \boldsymbol{n}_0 = \frac{\partial J}{\partial x} \cdot \boldsymbol{i} + \frac{\partial J}{\partial y} \cdot \boldsymbol{j} + \frac{\partial J}{\partial z} \cdot \boldsymbol{k} \tag{1}$$

where $\boldsymbol{n}_0$ is a universal unit vector $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are unit vectors in orthogonal axis system. ***Temperature gradient*** is a vector quantity, similarly as a thermal field intensity $\boldsymbol{E}_J$, that is

$$\boldsymbol{E}_J = -\operatorname{grad} J \tag{2}$$

In scalar thermal field the contour integral of thermal field intensity through some closed curve is always equals to zero, i.e.

$$\int_l \boldsymbol{E}_J \cdot \mathrm{d}\,l = \int_l \left(-\,grad\ J\right) \cdot \mathrm{d}\,l = 0 \tag{3}$$

In contrast, the integral of thermal field intensity only through some part of a closed curve (between two points of a curve) is non-equal to zero:

$$\int_l \boldsymbol{E}_J \cdot \mathrm{d}\,l = \int_l \left(-\,grad\ J\right) \cdot \mathrm{d}\,l = J_1 - J_2 = \Delta J \tag{4}$$

but, it is equal to temperature difference ΔJ between considered points. Implication of this difference ΔJ is heat transfer ***in direction of thermal field intensity***, eventually in direction of negative temperature gradient, thence from higher potential level to lower potential level

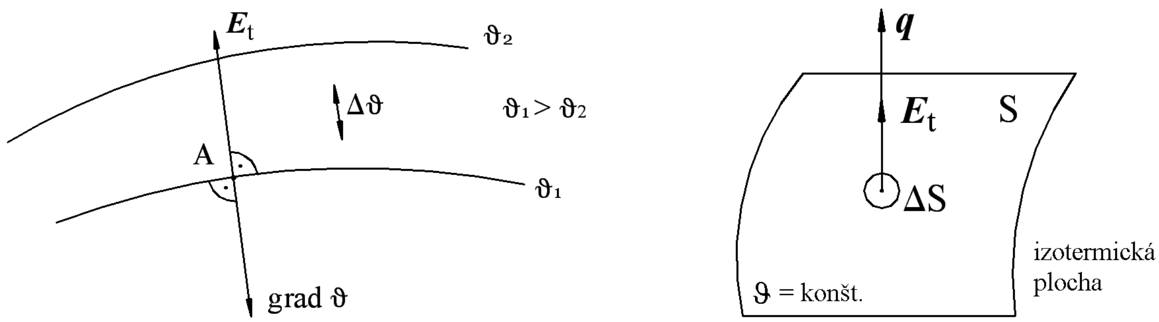
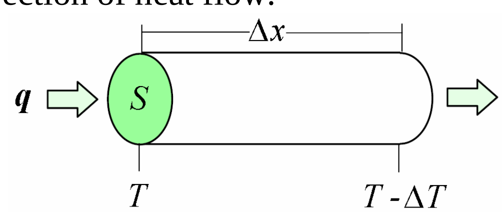


Illustration of temperature gradient and heat flux

Conduction heat transfer

Fourier's law:

The heat transfer rate in a solid is proportional to the temperature gradient and the cross-sectional area normal to the direction of heat flow.



Temperature gradient

$$\frac{(T-\Delta T)-T}{\Delta x} = -\frac{\Delta T}{\Delta x} \approx -\frac{\partial T}{\partial x} \tag{5}$$

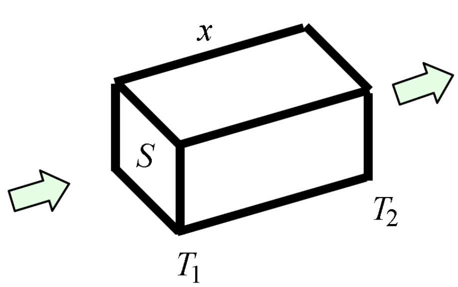
or

$$\Phi = -l \cdot S \cdot \frac{\partial T}{\partial x} \tag{6}$$

where l is the materials conductivity. (This generally varies with temperature, but the variation can be small, over a significant range of temperatures, for some common materials.) [W.m^{-1}.K^{-1}]

For ***steady*** 1-D heat flow the above equation (6) becomes:

$$\Phi = -l \cdot S \cdot \frac{(T_1 - T_2)}{x} \tag{7}$$



If we re-arrange the equation (7), as shown below, it can be seen to be directly analogous to ***Ohm's Law***:

$$T_1 - T_2 = \Phi \cdot \frac{x}{l \cdot S} \tag{8}$$

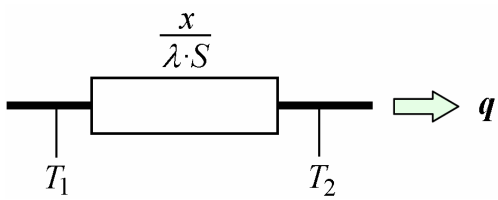
$$U = I \cdot R \tag{9}$$

where expression $\frac{x}{l \cdot S}$ can be thought of as a thermal resistance

$T_1 - T_2$ is the thermal potential

F is the thermal "current"

The symbol R_J is often used for ***thermal resistance***.



If we want to know what is a temperature dependence at various places of a body (on area, on the line), we have to determine the shape of temperature function according to particular coordinates.

If we re-arrange the equation (6) we obtain:

$$\frac{\partial T}{\partial x} = -\frac{\Phi}{l \cdot S} = -\frac{\Phi'}{l} \tag{10}$$

For steady state, the right hand side is constant, and after differentiating (10) gives:

$$\frac{\partial^2 T}{\partial x^2} = 0 \tag{11}$$

Note: generally:

$$\nabla^2 T = 0 \tag{12}$$

i.e. the condition of steady state 1-D conduction constitutes a solution to the above ***second order differential equation***. It is a Laplace's equation, which is mathematical model of steady thermal field ***without internal source***, because temperature is changed only in space, not in time.

For plane slab:

Issuing from Laplace's equation (12) and by application to 1-D field yields to:

$$-l \cdot \frac{\partial^2 T}{\partial x^2} = 0 \tag{13}$$

By double integrating of (13) and setting of boundary conditions (in place $x = 0$, $T = T_1$ and $x = s$, $T = T_2$) we can get linear change of temperature function T changing through its thickness:

$$-l \cdot \frac{\partial^2 T}{\partial x^2} = 0 \qquad / \int (\ ) \mathrm{d}x$$

$$-l \cdot \frac{\partial T}{\partial x} = c_1 \qquad / \int (\ ) \mathrm{d}x$$

$$-l \cdot T = c_1 \cdot x + c_2 \qquad / \cdot \left(-\frac{1}{l} \right)$$

$$T = -\frac{c_1 \cdot x}{l} - \frac{c_2}{l}$$ / subst.: $x = 0$, $T = T_1$ a $x = s$, $T = T_2$

$x = 0$, $T = T_1$:

$$T_1 = -\frac{c_1 \cdot 0}{l} - \frac{c_2}{l} = -\frac{c_2}{l} \qquad \Rightarrow \qquad c_2 = -T_1 \cdot l$$

$x = s$, $T = T_2$:

$$T_2 = -\frac{c_1 \cdot s}{l} - \frac{c_2}{l} = -\frac{c_1 \cdot s}{l} - \frac{(-T_1 \cdot l)}{l} = -\frac{c_1 \cdot s}{l} + T_1 \qquad / \cdot l$$

$$T_2 \cdot l = -c_1 \cdot s + T_1 \cdot l \qquad / -(T_1 \cdot l)$$

$$T_2 \cdot l - T_1 \cdot l = -c_1 \cdot s \qquad / \cdot \left(\frac{1}{s}\right)$$

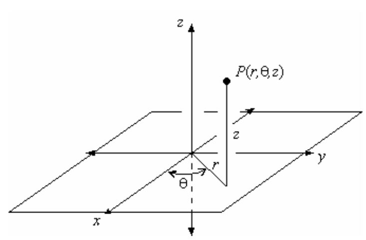
$$c_1 = -\frac{l}{s} \cdot (T_2 - T_1)$$

By substituting to the original equation we get the solution of temperature equation for 1-D field (plane slab):

$$T = -\frac{c_1 \cdot x}{l} - \frac{c_2}{l} = -\frac{\left[-\frac{l}{s} \cdot (T_2 - T_1)\right] \cdot x}{l} - \frac{(-T_1 \cdot l)}{l}$$

$$T = (T_2 - T_1) \cdot \frac{x}{s} + T_1$$

For cylindrical bar:



Heat transfer intensity in thermal field can be classified by quantities:

1. *Total amount of transferred heat Q* [J]
2. *Heat flow F* [W], that is identical with thermal field power

$$F = P_J = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{\mathrm{d}Q}{\mathrm{d}t} \qquad (6.6)$$

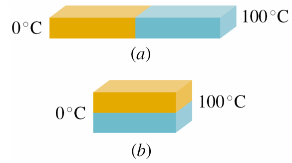
3. *Heat flux* $\boldsymbol{q}$ [W.m^{-2}]

$$\boldsymbol{q} = \lim_{\Delta S \to 0} \frac{\Delta F}{\Delta S} = \frac{\mathrm{d}F}{\mathrm{d}S} = \frac{\mathrm{d}Q}{\mathrm{d}S \cdot \mathrm{d}t} \qquad (6.7)$$

Necessary condition of heat transfer is inequality grad $J \neq 0$. In contrast that, in equality grad $J = 0$, the heat transfer in thermal field is not realized, field is isothermal. At last, thermal field can be without source or with source. In source field exists internal heat source, for example electrical.

Example 1

Two identical rectangular rods of metal are welded end to end and 10 J of heat is conducted (in a steady-state process) through the rods in 2 minutes. How long would it take for 10 J to be conducted through the rods if they are welded together lengthwise? (a = 1 cm, b = 2 cm, c = 4 cm).



Solution:

$$\Phi_{\text{endtoend}} = l \cdot S_1 \cdot \frac{\Delta J}{d_1} \qquad \Rightarrow \qquad \Phi_{\text{endtoend}} \cdot \frac{d_1}{S_1} = l \cdot \Delta J$$

$$\Phi_{\text{lengthwise}} = l \cdot S_2 \cdot \frac{\Delta J}{d_2} \qquad \Rightarrow \qquad \Phi_{\text{lengthwise}} \cdot \frac{d_2}{S_2} = l \cdot \Delta J$$

$$S_1 = a \cdot b; \qquad d_1 = c$$

$$S_2 = b \cdot \frac{c}{2}; \qquad d_2 = 2 \cdot a$$

$$\Phi_{\text{endtoend}} \cdot \frac{d_1}{S_1} = \Phi_{\text{lengthwise}} \cdot \frac{d_2}{S_2} \qquad \Rightarrow \qquad \Phi_{\text{lengthwise}} = \Phi_{\text{endtoend}} \cdot \frac{d_1}{S_1} \cdot \frac{S_2}{d_2} = \Phi_{\text{endtoend}} \cdot \frac{c}{a \cdot b} \cdot \frac{b \cdot \frac{c}{2}}{2 \cdot a}$$

$$\Phi_{\text{lengthwise}} = \Phi_{\text{endtoend}} \cdot \frac{c^2}{4 \cdot a^2} = \frac{10}{120} \cdot \frac{\left(4 \cdot 10^{-2}\right)^2}{4 \cdot \left(1 \cdot 10^{-2}\right)^2} = \frac{1}{12} \cdot 4 = \frac{1}{3}\ \text{J} \cdot \text{s}^{-1}$$

$$t = \frac{Q}{\Phi_{\text{lengthwise}}} = \frac{10}{\frac{1}{3}} = 30\ \text{s}$$

Example 2

Compute the rate of heat conduction through the following two storm doors 2 m high and 0,75 m wide.

a) One door is made with aluminum panels 1,5 mm thick and a 3,0-mm-thick glass panel that covers 75 % of its surface (the structural frame has a negligible area).
b) The second door is made entirely of white pine averaging 2,5 cm in thickness.

Take the temperature drop across each door to be 33 °C.

Solution:

a)

$$\Phi_{\text{Al}} = \frac{25}{100} \cdot \left(l \cdot S \cdot \frac{J_1 - J_2}{d} \right) = \frac{25}{100} \cdot \left[235 \cdot (2 \cdot 0{,}75) \cdot \frac{33}{1{,}5 \cdot 10^{-3}} \right] = \frac{7755000}{4} = 1938750\ \text{W}$$

$$\Phi_{\text{glass}} = l \cdot S \cdot \frac{J_1 - J_2}{d} = 1 \cdot (2 \cdot 0{,}75) \cdot \frac{33}{3 \cdot 10^{-3}} = 16500\ \text{W}$$

$$\Phi = \Phi_{\text{Al}} + \Phi_{\text{glass}} = 1938750 + 16500 = 1955250\ \text{W}$$

b)

$$\Phi_{\text{wood}} = l \cdot S \cdot \frac{J_1 - J_2}{d} = 0{,}11 \cdot (2 \cdot 0{,}75) \cdot \frac{33}{2{,}5 \cdot 10^{-2}} = 217{,}8\ \text{W}$$

Example 3

A large cylindrical water tank with a bottom 1,7 m in diameter is made of iron boiler plate 5,2 mm thick. As the water is being heated, the gas burner underneath is able to maintain a temperature difference of 2,3 °C between the top and bottom surfaces of the bottom plate. How much heat is conducted through that plate in 5 minutes? (Iron has a thermal conductivity of 67 W.m^{-1}.K^{-1}.)

Solution:

$$\Phi_1 = l \cdot S \cdot \frac{J_1 - J_2}{d} = 67 \cdot \left[p \cdot \left(\frac{1{,}7}{2} \right)^2 \right] \cdot \frac{2{,}3}{5{,}2 \cdot 10^{-3}} = 67264{,}67 \text{ W}$$

$$Q = \Phi_1 \cdot t = 67264{,}67 \cdot 5 \cdot 60 = 20179401 \text{ W}$$

Example 4

a) What is the rate of heat loss in watts per square meter through a glass window 3 mm thick if the outside temperature is –20 °C and the inside temperature is 22 °C?

b) A storm window is installed having the same thickness of glass but with an air gap of 7,5 cm between the two windows. What will be the corresponding rate of heat loss assuming that conduction is the only important heat-loss mechanism?

Solution:

a) $$q = \frac{\Phi}{S} = l \cdot \frac{J_1 - J_2}{d} = 1 \cdot \frac{22 - (-20)}{3 \cdot 10^{-3}} = 14000 \text{ W} \cdot \text{m}^{-2}$$

b) $$q = \frac{\Phi}{S} = l \cdot \frac{J_1 - J_2}{d} = \frac{J_1 - J_2}{\frac{d_{\text{glass}}}{l_{\text{glass}}} + \frac{d_{\text{air}}}{l_{\text{air}}} + \frac{d_{\text{glass}}}{l_{\text{glass}}}} = \frac{22 - (-20)}{\frac{3 \cdot 10^{-3}}{1} + \frac{7{,}5 \cdot 10^{-3}}{0{,}026} + \frac{3 \cdot 10^{-3}}{1}} = 142{,}6 \text{ W} \cdot \text{m}^{-2}$$

Example 5

The average rate at which heat is conducted through the surface of the Earth is 54 mW.m^{-2}. And the average thermal conductivity of the near-surface rocks is 2,5 W.m^{-1}.K^{-1}. Assuming a surface temperature of 10 °C, what should be the temperature at a depth of 35 km (near the base of the crust)? Ignore the heat generated by the presence of radioactive elements.

Solution:

$$q = \frac{\Phi}{S} = l \cdot \frac{J_1 - J_2}{d} \quad \Rightarrow \quad J_1 = q \cdot \frac{d}{l} + J_2 = 54 \cdot 10^{-3} \cdot \frac{35 \cdot 10^3}{2{,}5} + 10 = 766 \text{ °C}$$

Example 6

The thermal conductivity of Pyrex glass at 0 °C is 2,9.10^{-3} cal/(cm.°C.s)

a) Express this in units of SI.

b) What is the thermal resistance value R_J for 25-in. sheet of such glass?

Solution:

a) $1{,}2 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$

b) $$R = \frac{l}{l} = \frac{0{,}25 \cdot (2{,}54 \cdot 10^{-2})}{1{,}2} = 5{,}3 \cdot 10^{-3} \text{ m}^2 \cdot \text{K} \cdot \text{W}^{-1}$$

Example 7

a) Calculate the rate at which body heat is conducted through the clothing of a skier in a steady-state process, given the following data: the body surface area is 1,8 m^2 and the clothing is 1 cm thick; the skin surface temperature is 33 °C, whereas the outer surface of the clothing is at 1 °C; the thermal conductivity of the clothing is 0,04 W/(m.K).

b) How would the answer to a) change if after a fall, the skier's clothes became soaked with water? Assume that the thermal conductivity of water is 0,6 W.m^{-1}.K^{-1}.

Solution:

a) $$\Phi = l \cdot S \cdot \frac{J_1 - J_2}{d} = 0{,}04 \cdot 1{,}8 \cdot \frac{33-1}{1 \cdot 10^{-2}} = 230{,}4\ \mathrm{J \cdot s^{-1}}$$

b) $$\Phi_{\mathrm{withwater}} = l \cdot S \cdot \frac{J_1 - J_2}{d} = 0{,}6 \cdot 1{,}8 \cdot \frac{33-1}{1 \cdot 10^{-2}} = 3456\ \mathrm{J \cdot s^{-1}}$$

what is approximately 15-times more than dry clothing.

Example 8

A cylindrical copper rod of length 1,2 m and cross-sectional area 4,8 cm^2 is insulated to prevent heat loss through its surface. The ends are maintained at a temperature difference of 100 °C by having one end in a water-ice mixture and the other in boiling water and steam.

a) Find the rate at which heat is conducted along the rod.

b) Find the rate at which ice melts at the cold end.

Solution:

a) $$\Phi = l \cdot S \cdot \frac{J_1 - J_2}{d} = 401 \cdot \left(4{,}8 \cdot 10^{-4}\right) \cdot \frac{100}{1{,}2} = 16{,}04\ \mathrm{J \cdot s^{-1}}$$

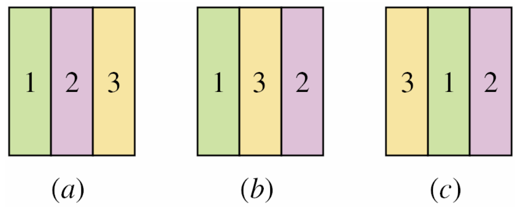
b) $$\left|\frac{\mathrm{d}\,m}{\mathrm{d}\,t}\right| = \frac{\Phi}{L_{\mathrm{melting}}} = \frac{16\ \mathrm{J \cdot s^{-1}}}{333\ \mathrm{kJ \cdot kg^{-1}}} = 0{,}048\ \mathrm{g \cdot s^{-1}}$$

Example 9

At the picture are three various arrangements of materials 1, 2 and 3 that create the wall. Their thermal conductivities are $k_1 > k_2 > k_3$. Left side of the wall is about 20 °C hotter than right side. Order walls incrementally according to:

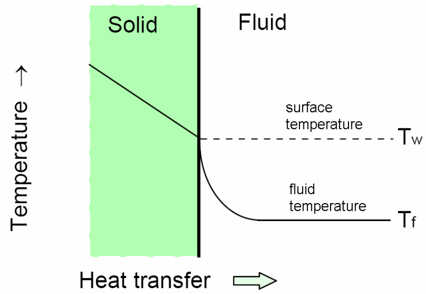
a) heat flux through the wall,

b) thermal loss in layer 1.



Convection heat transfer

Heat transfer by convection occurs within a fluid or more typically at the interface between a solid boundary and a fluid.



The fluid is normally moving relative to the wall because the temperature changes causes density changes which cause the fluid to rise (elasticity) or sink (negative elasticity). Heat transfer to a fluid which moves because of the heat transfer is called ***natural convection***.
Alternatively the fluid flow can be caused by a fan or a pump. Heat transfer to a fluid under these circumstances is termed ***forced convection***. Both are important.

Because the temperature changes very rapidly close to surface (in the boundary layer) x and l cannot be used.

Newton's law of cooling

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings). In the other way, the heat transfer rate is proportional to the surface area and the temperature difference between the surface and the fluid.

$$\Phi \propto S \cdot (T_{\mathrm{w}} - T_{\mathrm{f}})$$

The heat transfer rate, area, and temperature difference are correlated by the ***surface*** (or ***film***) ***heat transfer coefficient*** a.

$$\Phi = q \cdot S = a \cdot S \cdot (T_{\mathrm{w}} - T_{\mathrm{f}})$$

This is the fundamental equation for heat convection. However, it is 'illusive' simple J

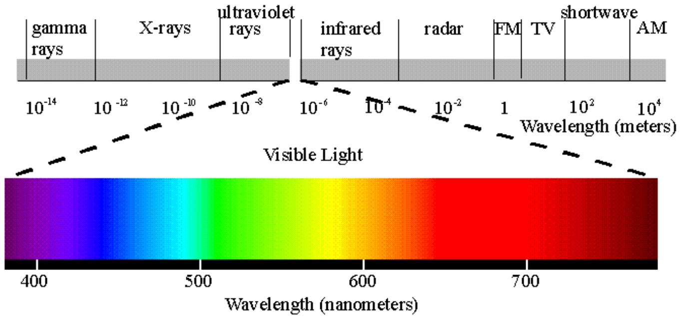
Although a may be easily measured (in many cases) it is not easy to predict.
It depends on:

- the type of flow (laminar or turbulent)
- viscosity of the fluid
- thermal conductivity of the fluid
- velocity over the surface
- geometry of the surface
- specific heat capacity of the fluid
- expansion coefficient of the fluid (natural convection especially)
- density of the fluid (natural convection especially)

For some common situations, equations have been derived experimentally (empirical) with the help of 'dimensional analysis'. Dimensional analysis reveals certain non-dimensional parametric groups which arise from the physical circumstances.

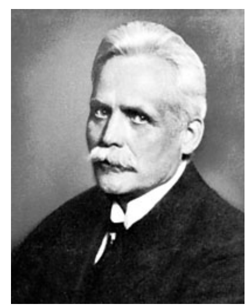
Radiation

Thermal radiation is generated when heat from the movement of charged particles within atoms is converted to electromagnetic radiation. Thermal radiation is emitted by every body of temperature higher than 0 K. Thermal radiation of a body is electromagnetic waving of various wavelengths in dependence on body temperature and body structure. Close to temperatures of 500 °C thermal radiation is mainly infrared. When temperature is increased in the range of visible light the radiated energy is also rising.



Wien's displacement law – is a law that states that there is an inverse relationship between the wavelength of the peak of the emission of a black body and its temperature.

$$l = \frac{b}{T} \qquad [\mathrm{m; m.K, K}] \qquad (1)$$



where l is the peak wavelength [m]

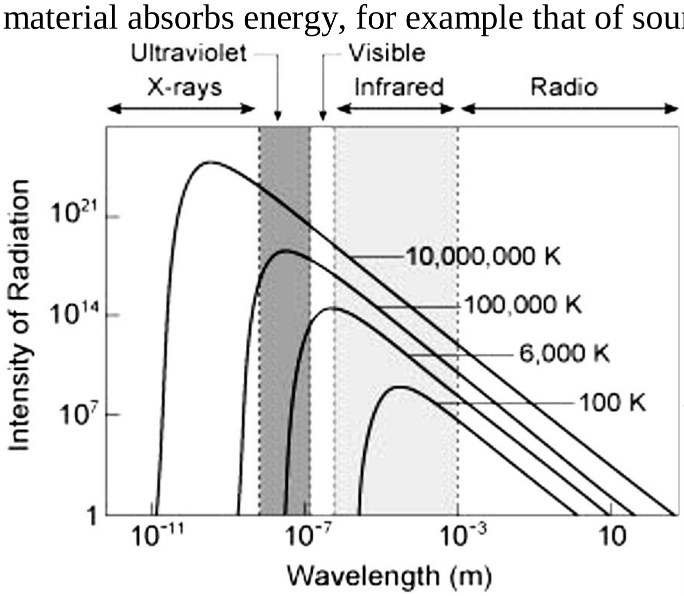
b is a constant of proportionality, called Wien's displacement constant; b = 2,898.10^{-3} m.K

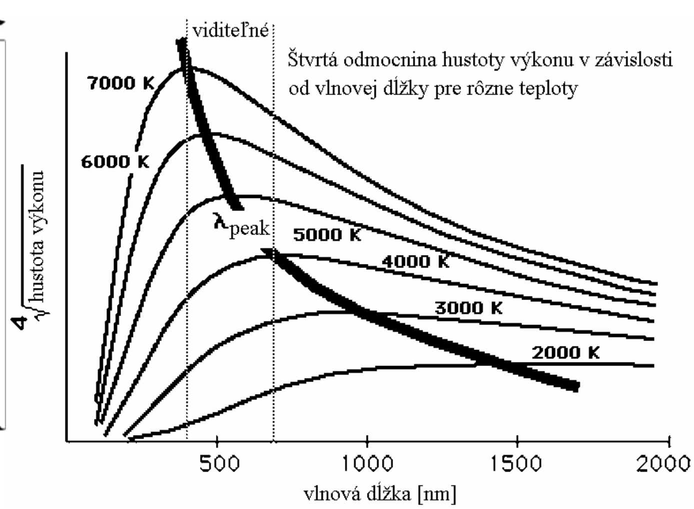
T is the temperature of the blackbody [K]

A black body – is an object that absorbs all electromagnetic radiation that falls onto it. No radiation passes through it and none is reflected.

A grey body – is defined as a body with *constant emissivity* over all wavelengths and temperatures. Such an ideal body does not exist in practice but the assumption is a good approximation for many objects used in engineering. The absorption coefficient of a grey body is $k < 1$.

The absorption coefficient k – is a property of a material. It defines the extent to which a material absorbs energy, for example that of sound waves or electromagnetic radiation.





The ***Stefan-Boltzmann law*** (published in 1879 by Ludwig Boltzmann and Josef Stefan) – states that the *total energy* radiated per unit surface area of a black body in unit time (known variously as the *black-body irradiance*, *energy flux density*, *radiant flux*, or the *emissive power*), is directly proportional to the fourth power of the black body's thermodynamic temperature T:

$$q = s \cdot e \cdot T^4 \qquad [\mathrm{W.m^{-2}; J.s^{-1}.m^{-2}.K^{-4}, –, K}] \qquad (2)$$

where q energy flux density [W.m^{-2}]

e emissivity of the blackbody [–]

s Stefan-Boltzmann's constant

$$s = \frac{2 \cdot p^5 \cdot k^4}{15 \cdot c^2 \cdot h^3} = 5{,}6704 \cdot 10^{-8}\ \mathrm{J \cdot s^{-1} \cdot m^{-2} \cdot K^{-4}}$$

where k Boltzmann's constant; k = (1,380658 ± 0,000012).10^{-23} J.K^{-1}

c speed of light in a vacuum; c = 299 792 458 m.s^{-1}

h Planck's constant; h = 6,62607.10^{-34} J.s

T thermodynamic temperature [K]

Radiation between two gray surface bodies

Thermal radiation (infra-red radiation) is spread by speed of light, but its wavelength is higher. Analogous to visible radiation also infra-red radiation is reflected by glossy furnishes and absorbed by mat and rough surfaces. Surfaces with high ***reflexivity*** has low ***emissivity*** and.

Material	***r*** (%)	***e***
glossy silver	99	0,01
glossy copper	98	0,02
glass	8	0,92
paper	10	0,90
concrete	11	0,89
paints and enamels	8÷15	0,88
planed wood	20	0,80

The energetic balance of radiating energy to the body surface consists of energy elements through the surface passed:

- **a**bsorbed
- **r**eflected
- released (**d**iffused)

Total fell radiation is divided into:

$$Q^* = Q^*_A + Q^*_R + Q^*_D \quad (3) \qquad \text{or} \qquad \frac{Q^*_A}{Q^*} + \frac{Q^*_R}{Q^*} + \frac{Q^*_D}{Q^*} = A + R + D = 1 \quad (4)$$

Size of coefficient is theoretically in the range of ⟨0, 1⟩. For black body (A = 1, R = D = 0), white body (R = 1, A = D = 0) and absolutely released body (D = 1, A = R = 0). These bodies are ideal, for real solid bodies A < 1, R < 1, D = 0, thence

$$A + R = 1 \qquad (5)$$

Real solid bodies with property (5) are named ***grey***.

Example 1

A ball with radius 0,5 m and with temperature 27 °C has emissivity 0,85 is in surrounding with temperature of 77 °C.

a) What is radiated flow (power) from a ball?
b) What is absorbed flow to a ball?
c) What is total radiated flow of a ball?

Solution:

a) Ball temperature:

$T = 273{,}15 + 27 = 300{,}15\ \mathrm{K}$

Radiated flow:

$$\Phi_{\mathrm{radiation}} = s \cdot e \cdot S \cdot T^4 = \left(5{,}67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}\right) \cdot 0{,}85 \cdot \left[4 \cdot p \cdot (0{,}5\ \mathrm{m})^2\right] \cdot (300{,}15\ \mathrm{K})^4 = 1{,}23 \cdot 10^3\ \mathrm{W}$$

b) Surrounding temperature:

$T_{\mathrm{sur}} = 273{,}15 + 77 = 350{,}15\ \mathrm{K}$

Absorbed flow:

$$\Phi_{\mathrm{absorption}} = s \cdot e \cdot S \cdot T_{\mathrm{sur}}^4 = \left(5{,}67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}\right) \cdot 0{,}85 \cdot \left[4 \cdot p \cdot (0{,}5\ \mathrm{m})^2\right] \cdot (350{,}15\ \mathrm{K})^4 = 2{,}28 \cdot 10^3\ \mathrm{W}$$

c) Total radiated flow of a ball:

$$\Phi_{\mathrm{total}} = \Phi_{\mathrm{absorption}} - \Phi_{\mathrm{radiation}} = 2{,}28 \cdot 10^3\ \mathrm{W} - 1{,}23 \cdot 10^3\ \mathrm{W} = 1{,}05 \cdot 10^3\ \mathrm{W}$$

Example 2

A cube with edge size 6.10^{-6} m, emissivity 0,75 and with temperature –100 °C is inserted to surrounding of –150 °C. What thermal flow exchange cube with its surrounding?

Solution:

A cube has 6 faces, and every face has area $(6.10^{-6})^2$ m^2. Temperatures in unit of °C we convert into Kelvin and then:

$$\Phi = s \cdot e \cdot S \cdot \left(T_{\mathrm{sur}}^4 - T^4\right) =$$

$$= \left(5{,}67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}\right) \cdot 0{,}75 \cdot \left(2{,}16 \cdot 10^{-10}\,\mathrm{m}^2\right) \cdot \left[(123{,}15\ \mathrm{K})^4 - (173{,}15\ \mathrm{K})^4\right] = -6{,}1 \cdot 10^{-9}\ \mathrm{W}$$

Example 3

A cylinder with radius $r_1 = 2{,}5$ cm and length $h_1 = 5$ cm has emissivity 0,85 and temperature 30 °C. It is hung in surrounding with temperature 50 °C.

a) What is total radiated flow (power) of a cylinder F_1?
b) What is new flow F_2, when we change radius of a cylinder to $r_2 = 0{,}5$ cm and volume is the same?
c) What is relation between F_2/F_1?

Solution:

a) Cylinder area can be specified as follow:

$$S_1 = 2 \cdot \left(p \cdot r_1^2\right) + 2 \cdot p \cdot r_1 \cdot h_1 = 2 \cdot p \cdot \left(2{,}5 \cdot 10^{-2}\ \mathrm{m}\right)^2 + 2 \cdot p \cdot \left(2{,}5 \cdot 10^{-2}\ \mathrm{m}\right) \cdot \left(5 \cdot 10^{-2}\ \mathrm{m}\right) = 1{,}18 \cdot 10^{-2}\ \mathrm{m}^2$$

Cylinder temperature is:

$T_1 = 273 + 30 = 303\ \mathrm{K}$

and surrounding temperature:

$T_{sur} = 273 + 50 = 323\ \mathrm{K}$

Substituting into Stefan-Boltzmann's equation for gray bodies:

$$\Phi_1 = s \cdot e \cdot S_1 \cdot \left(T_{sur}^4 - T^4\right) =$$

$$= \left(5{,}67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}\right) \cdot 0{,}85 \cdot \left(1{,}18 \cdot 10^{-2}\,\mathrm{m}^2\right) \cdot \left[(323\ \mathrm{K})^4 - (303\ \mathrm{K})^4\right] = 1{,}39\ \mathrm{W}$$

b) Let new cylinder length is h_2. As volume V is fixed, then must be valid equality $V = p \cdot r_1^2 \cdot h_1 = p \cdot r_2^2 \cdot h_2$. Expressing h_2:

$$h_2 = \left(\frac{r_1}{r_2}\right)^2 \cdot h_1 = \left(\frac{2{,}5\ \mathrm{cm}}{0{,}5\ \mathrm{cm}}\right)^2 \cdot (5\ \mathrm{cm}) = 125\ \mathrm{cm} = 1{,}25\ \mathrm{m}$$

New cylinder area S_2 is:

$$S_2 = 2 \cdot \left(p \cdot r_2^2\right) + 2 \cdot p \cdot r_2 \cdot h_2 = 2 \cdot p \cdot \left(0{,}5 \cdot 10^{-2}\ \mathrm{m}\right)^2 + 2 \cdot p \cdot \left(0{,}5 \cdot 10^{-2}\ \mathrm{m}\right) \cdot (1{,}25\ \mathrm{m}) = 3{,}94 \cdot 10^{-2}\ \mathrm{m}^2$$

Radiated flow (power) of a cylinder F_2:

$$\Phi_2 = s \cdot e \cdot S_2 \cdot \left(T_{sur}^4 - T^4\right) =$$

$$= \left(5{,}67 \cdot 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}\right) \cdot 0{,}85 \cdot \left(3{,}94 \cdot 10^{-2}\,\mathrm{m}^2\right) \cdot \left[(323\ \mathrm{K})^4 - (303\ \mathrm{K})^4\right] = 4{,}663\ \mathrm{W}$$

c) relation between F_2/F_1:

$$\frac{\Phi_2}{\Phi_1} = \frac{S_2}{S_1} = \frac{3{,}94 \cdot 10^{-2}\ \mathrm{m}^2}{1{,}18 \cdot 10^{-2}\ \mathrm{m}^2} = 3{,}3$$