Temperature is one of the basic seven fundamental physical quantities. The scale of thermodynamic temperature is defined by assigning to the triple point of water a temperature of 273,16 K. The SI unit of temperature (Kelvin) is then simply defined as 1/273,16 the temperature of the water triple point. The temperature of a system is related to the average energy of microscopic motions in the system. The basic unit of temperature is Kelvin. We also use other units for temperature: Celsius scales, Fahrenheit, Rankine, etc. Temperature is qualified by letter T [K] or J [°C]. It can be measured ***directly*** (by thermometer: gas thermometer, liquid thermometer, bimetal thermometer, thermocouple, thermostat (resistor, that has rezistivity changed by influencing of temperature)) or ***indirectly*** (pyrometer, thermograph).

The zeroth law of thermodynamics is a generalized statement about bodies in contact at thermal equilibrium and is the basis for the concept of temperature. The most common enunciation of the zeroth law of thermodynamics is: "*If two thermodynamic systems are in thermal equilibrium with a third, they are also in thermal equilibrium with each other.*" In other words, the zeroth law says that if considered a mathematical binary relation, thermal equilibrium is transitive.

Thermal expansion is the tendency of matter to increase in volume or pressure when heated. For liquids and solids the amount of expansion will normally vary depending on the ***material's coefficient of thermal expansion***. In general, there is considered continual proportion between quantity change ΔX and temperature change ΔT.

$$\Delta X = X_0 \cdot k \cdot \Delta T$$

where X_0 represents initial value of quantity X before temperature change, k is material's coefficient of thermal expansion, which is in units of [1/K] or [1/°C].

Linear thermal expansion

$$\Delta d = d_0 \cdot a \cdot \Delta J \qquad \text{or} \qquad \Delta d = d_0 \cdot a \cdot \Delta T \qquad [\mathrm{m}]$$

where a is material's coefficient of linear thermal expansion [1/°C]

Volume thermal expansion

$$\Delta V = V_0 \cdot b \cdot \Delta J \qquad \text{or} \qquad \Delta V = V_0 \cdot b \cdot \Delta T \qquad [\mathrm{m}^3]$$

where b is material's coefficient of volume thermal expansion [1/°C] ($b = 3 \cdot a$)

***Heat** Q* is the energy that is transferred between a system and its environment because of a temperature difference between them. It can be measured in joules [J], calories [cal], kilocalories [Cal or kcal], or British thermal units [Btu], with

$$1\,\mathrm{cal} = 3{,}969 \cdot 10^{-3}\ \mathrm{Btu} = 4{,}186\ \mathrm{J}$$

Generally, heat is a form of energy transfer associated with the different motions of atoms, molecules and other particles that comprise matter when it is hot and when it is cold. Heat can be transferred between objects by ***radiation***, ***conduction*** and ***convection***. Temperature, defined as the measure of an object to spontaneously give up energy, is used as a measure of the internal energy or enthalpy, which is the level of elementary motion giving rise to heat transfer. *Heat can only be transferred between objects, or areas within an object, with different temperatures* (as given by the zeroth law of thermodynamics), and then, in the absence of work, only *in the direction of the colder body* (as per the second law of thermodynamics).

Heat Capacity and ***Specific Heat***

If heat Q is added to an object with mass m, the temperature change $T_2 - T_1$ is related to Q by

$$Q = C \cdot (T_2 - T_1)$$

where C is heat capacity [J/K]

Heat capacity is a measurable physical quantity that characterizes the ability of a body to store heat as it changes in temperature. It is defined as the rate of change of temperature as heat is added to a body at the given conditions and state of the body.

Heat capacity is defined as quantity of heat in Joules, which is needed to add to body to increase its temperature by o 1 K (Kelvin), eventually o 1 °C (degree of Celsius).

$$C = \frac{\partial Q}{\partial T} \quad [\mathrm{J.K^{-1}}] \qquad \text{or} \qquad C = \lim_{\Delta T \to 0} \frac{Q}{\Delta T} \quad [\mathrm{J.K^{-1}}]$$

More often there are presented in tables ***specific heat capacity***, which is related to unit of mass (weight). It is denoted by small c. Unit of specific heat capacity is [J/(kg.K)], eventually [kJ/(kg.K)].

$$c = \frac{C}{m} \quad [\mathrm{J/(kg.K)}]$$

where m is mass of the body [kg]

A heat added to a body increases temperature by $J_2 - J_1$. This dependence is expressed by relation:

$$Q = C \cdot (J_2 - J_1) \quad [\mathrm{J}]$$

where C is heat capacity of a body

If m is a mass of a body, then

$$Q = c \cdot m \cdot (J_2 - J_1) \quad [\mathrm{J}]$$

where c is specific heat capacity of material [J/(kg.K)]

Question:

Some amount of a heat Q can heat 1 g of material A by 3 °C and 1 g of material B by 4 °C. Which one of these materials has greater specific heat capacity?

Heat per mass unit, that is added to material, needed for change of its state, is named ***heat of transformation***, eventually ***latent heat*** L.

$$Q = L \cdot m$$

Most commonly we deal wit the ***heat of vaporization*** L_v, what is amount of energy pre unit mass that must be added to vaporize a liquid or that must be removed to condense a gas. The ***heat of fusion*** L_f is the amount of energy per unit mass that must be added to melt a solid or that must be removed to freeze liquid.

First law of thermodynamics

The principle of conservation of energy for a sample of material exchanging energy with its surroundings by means of work and heat is expressed in the ***first law of thermodynamics***, which may assume either of the forms:

formulation 1: Every physical system has a static quantity named internal energy (U), which can be changed only by interaction of energy with surroundings (volume work, thermal exchange)

formulation 2: $\Delta U = \Delta Q - \Delta(W_p + W_k) = \Delta Q - \Delta W$ [J]

where ΔU internal energy change of a system

ΔQ heat change of a system (+ means addition, – means subtraction)

$\Delta(W_p + W_k)$ performed/spent volume work (+ means, that a system made a work, – means, that a system spent a work)

formulation 3: It is not possible to design ***perpetual motion of the first kind***, that produces strictly more energy than it uses, thus violating the law of conservation of energy. Over-unity devices, that is, devices with a thermodynamic efficiency greater than 1.0 (unity, or 100%), are perpetual motion machines of this kind.

Application of first law of thermodynamics

First law of thermodynamics can be applied into actions executed in closed systems. ***We establish a consensus***: volume work W will be denoted overall, not as a difference between performed and spent work. The sign of a work can show the type of a work:

$W > 0$ work was spent by a system (or work was added to system)

$W < 0$ work was performed by a system

First law of thermodynamics can be also used in followed special causes:

Adiabatic process $Q = 0$: $Q = 0$, $\Delta U = -W$,

Isochoric process $\Delta V = 0$: $W = 0$, $\Delta U = Q$,

Isothermal process T = const.: $\Delta U = 0$, $Q = -W$,

Isobaric process p = const.: $\Delta U = Q + W$ ($W = -p \times \Delta V$, $Q = \Delta U - W$)

Heat transfer

Power F, which is transferred by ***conduction*** through the solid material with temperatures of side walls J_2 and J_1, is

$$\Phi = \frac{Q}{t} = l \cdot S \cdot \frac{J_2 - J_1}{d} \quad \text{[W]} \quad \text{eventually} \quad \Phi = \frac{Q}{t} = l \cdot S \cdot \frac{T_2 - T_1}{d} \quad \text{[W]}$$

where S board surface [m^2]

d board thickness [m]

l thermal conductivity factor of material [W/(m.K)]

Convection is the internal movement of currents within fluids (i.e. liquids and gases). It cannot occur in solids due to the atoms not being able to flow freely. Differential heating of fluids may cause convection due to variations in density due to a transfer of heat and expansion. This type of heat-driven convection is sometimes referred to as "***natural heat convection***" to distinguish it from forced heat convection where transfer of heat due to movement in the fluid from other forces.

Radiation is the process of emitting energy in the form of waves or particles. Radiation is, essentially, energy traveling through space. Power F_{r}, that is radiated from a body by thermal radiation is equal to

$$\Phi_{\mathrm{r}} = s \cdot e \cdot S \cdot T^4 \qquad [\mathrm{W}]$$

where s is Stefan-Boltzmann’s constant; s = 5,6703×10^{-8} W/(m^2.K^4)
e is emisivity of material surface [-]
S is surface of material [m^2]
T is surface temperature of material [K]

Power F_{a}, which is absorbed by a body by thermal radiation from its surroundings, is in constant ambient temperature T_0 [K] equals to

$$\Phi_{\mathrm{a}} = s \cdot e \cdot S \cdot T_0^4 \qquad [\mathrm{W}]$$

Exercise 1

Materials A, B and C are solid materials at temperature of their fusion. Material A needs 200 J for melting of 4 kg. Material B needs 300 J for melting of 5 kg and material C needs 300 J for melting of 6 kg. Order them in descending order according to their specific heat of fusion.

Exercise 2

Size of a glass window at temperature 10 °C is exactly 20 cm × 30 cm. What is a surface difference at temperature 40 °C?

Solution:

$$\Delta S = S \cdot (2 \cdot a) \cdot (J_1 - J_2) = 0{,}2 \cdot 0{,}3 \cdot \left(2 \cdot 9 \cdot 10^{-6}\right) \cdot (40 - 10) = 3{,}24 \cdot 10^{-5}\ \mathrm{m}^2$$

Exercise 3

A pendulum clock with a pendulum made of brass is designed to keep accurate time at 20 °C. What will be the error, in seconds per hour, if the clock operates at 0 °C? ($a = 19 \cdot 10^{-6}\ /\mathrm{°C}$)

Exercise 4

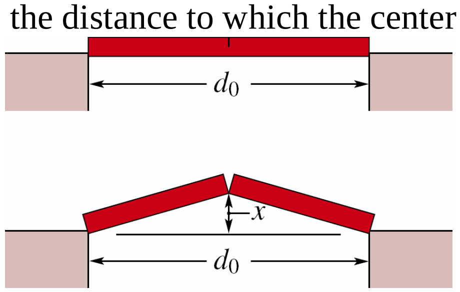
What is the volume of a lead ball at 30 °C, if its volume at 60 °C is 50 cm^3?

Solution:

$$V = V_0 \cdot (1 + b \cdot \Delta J) = V_0 \cdot (1 + 3 \cdot a \cdot \Delta J) = \left(50\ \mathrm{cm}^3\right) \cdot \left[1 + 3 \cdot \left(29 \cdot 10^{-6}\ /\mathrm{°C}\right) \cdot (30\ \mathrm{°C} - 60\ \mathrm{°C})\right] = 49{,}87\ \mathrm{cm}^3$$

Exercise 5

As a result of a temperature rise of 32°C, a bar with a crack at its center buckles upward, as shown in figure bellow. If the fixed distance $d_0 = 3{,}77$ m and the coefficient of linear expansion is 25.10^{-6} 1/°C, find x, the distance to which the center rises.



Solution:

We consider only half of a bar. Its half length is $l_0 = d_0 / 2$ and its length after temperature rising is $l = l_0 + a \cdot l_0 \cdot \Delta J$. Original length of half-bar l_0, its new length l and distance x create right-angled triangle. Using Pythagoras' theorem: $x = l^2 - l_0^2 = l_0^2 \cdot (1 + a \cdot \Delta J)^2 - l_0^2$. After expressing elements of term $(1 + a \cdot \Delta J)^2$ we discover, that term $(a \cdot \Delta J)^2$ compared to elements $1 + 2 \cdot a \cdot \Delta J$ is small and we can it neglect. After it we can get

$$x^2 = l_0^2 + 2 \cdot l_0^2 \cdot a \cdot \Delta J - l_0^2 = 2 \cdot l_0^2 \cdot a \cdot \Delta J$$

and

$$x = l_0 \cdot \sqrt{2 \cdot a \cdot \Delta J} = \frac{3{,}77\ \mathrm{m}}{2} \cdot \sqrt{2 \cdot \left(25 \cdot 10^{-6}\ /\mathrm{°C}\right) \cdot (32\ \mathrm{°C})} = 7{,}5 \cdot 10^{-2}\ \mathrm{m}.$$

Exercise 6

A diet doctor encourages dieting by drinking ice water. His theory is that the body must burn off enough fat to raise the temperature of the water from (0 °C) to the body temperature of (37 °C). How many liters of ice water would have to be consumed to burn off 454 g (about 1 lb) of fat, assuming that this requires 3500 kcal? Why is it not advisable to follow this diet?

Solution:

$$Q_{\text{fat}} = 3500 \cdot 10^{3} \text{ cal} \cdot 4{,}186 = 1{,}4651 \cdot 10^{7} \text{ J}$$

$$Q_{\text{water}} = Q_{\text{fat}} = m_{\text{water}} \cdot c_{\text{water}} \cdot \Delta J$$

Then:

$$m_{\text{water}} = \frac{Q_{\text{fat}}}{c \cdot \Delta J} = \frac{1{,}4651 \cdot 10^{7}}{4186 \cdot 37} = 94{,}59 \text{ kg}.$$

Exercise 7

A room is lighted by four 100-W incandescent light bulbs. (The power of 100 W is the rate at which a bulb converts electrical energy into heat and visible light. Assuming that 90 % of the energy is converted to heat, how much he at is added to the room in 1 hour?

Exercise 8 (Continuation of example 7 **J**)

An energetic athlete dissipates all the energy in a diet of 4000 kcal/day. If he were to release this energy at a steady rate, how would this conversion of energy compare with that of a 100-W bulb? (The power of 100 W is the rate at which a bulb converts electrical energy into heat and visible light.)

Exercise 9

The specific heat of a substance varies with temperature according to $c = 0{,}2 + 0{,}14 \cdot J + 0{,}023 \cdot J^2$. (Temperature is in degree of Celsius and c is in cal/(g·K)). Find the heat required to raise the temperature of 2 g of this substance from 5 °C to 15 °C.

Solution:

$$Q = \int_{J_1}^{J_2} c \cdot m \cdot \mathrm{d}J = m \cdot \int_{J_1}^{J_2} c \cdot \mathrm{d}J = 2 \cdot \int_{5\,°\text{C}}^{15\,°\text{C}} \left(0{,}2 + 0{,}14 \cdot J + 0{,}023 \cdot J^{2}\right) \cdot \mathrm{d}J =$$

$$= 2 \cdot \left[0{,}2 \cdot J + 0{,}07 \cdot J^{2} + 0{,}00767 \cdot J^{3}\right]_{5}^{15} \text{ cal} = 82 \text{ cal}$$

Exercise 10

A chef, up on awaking one morning to find his stove out of order, decides to boil the water for his wife's coffee by shaking it in a thermos flask. Suppose that he uses 500 cm^3 of tap water at 20 °C, and that the water falls 30 cm each shake, the chef making 30 shakes each minute. Neglecting any loss of thermal energy by the flask, how long uses 500 cm^3 of tap water at 20 °C, and that the he shake the flask before the water boils?

Solution:

$E_{\text{p}} = Q$

$$t \cdot n \cdot m \cdot g \cdot h = c \cdot m \cdot \left(J_2 - J_1\right)$$

$$t = \frac{Q}{n \cdot m \cdot g \cdot h} = \frac{c \cdot m \cdot \left(J_2 - J_1\right)}{\frac{30 \text{ taps}}{1 \text{ minute}} \cdot m \cdot g \cdot h} = \frac{4186 \cdot 0{,}5 \cdot (100 - 20)}{\frac{30 \text{ taps}}{1 \text{ minute}} \cdot 0{,}5 \cdot 9{,}81 \cdot 0{,}3} = 3792 \text{ min} = 63{,}22 \text{ hours} = 2 \text{ days}\, 15 \text{ hours}$$

Exercise 11

A candy bar has a marked nutritional value of 350 kcal. How many kilowatt-hours of energy will it deliver to the body as it is digested?

Solution:

This amount of energy would keep a 100-W light bulb burning for 4,1 h. To burn up this much energy by exercise, a person would have to jog about 5-6 km.

A generous daily human diet corresponds to about 3,5 kW·h per day, which represents the absolute maximum amount of work that a human can do in one day. In Slovakia, this amount of energy can be purchased for perhaps 4,50 Sk/(kW·h) × 3,5 kW·h = 15,80 Sk.

Solution:

$$E = \left(350 \cdot 10^3\ \mathrm{cal}\right) \cdot \left(4{,}19\ \mathrm{J/kg}\right) = \left(1{,}466 \cdot 10^6\ \mathrm{J}\right) \cdot \left(1\ \mathrm{W \cdot s/J}\right) \cdot \left(1\ \mathrm{h} / 3600\ \mathrm{s}\right) \cdot \left(1\ \mathrm{kW} / 1000\ \mathrm{W}\right) = 0{,}407\ \mathrm{kW \cdot h}$$

Exercise 12

A copper slug whose mass m_{Cu} = 75 g is heated in a laboratory oven to a temperature J = 312 °C. The slug is then dropped into a glass beaker containing amass m_{v} = 220 g of water. The effective heat capacity C_{n} of the beaker is 45 cal/K. The initial temperature J_1 of the water and the beaker is 12 °C. What is the final temperature J_2 of the slug, the beaker, and the water?

Solution:

Let us take as our system the water + beaker + cooper slug. No heat enters or leaves this system, so the algebraic sum of the internal heat transfers that occur must be zero. There are three such transfers:

for the water: $Q_{\mathrm{v}} = m_{\mathrm{v}} \cdot c_{\mathrm{v}} \cdot \left(J_2 - J_1\right)$;

for the beaker: $Q_{\mathrm{n}} = C_{\mathrm{n}} \cdot \left(J_2 - J_1\right)$;

for the copper: $Q_{\mathrm{Cu}} = m_{\mathrm{Cu}} \cdot c_{\mathrm{Cu}} \cdot \left(J_2 - J\right)$.

The temperature difference is written – in all three cases – as the final temperature minus the initial temperature. We do this even though we realize that Q_{v} and Q_{n} are positive (indicating that heat is added to the initially cool water and beaker) and that Q_{Cu} is negative (indicating that heat is released by the initially hot copper slug). From what we have said above, we must have

$$Q_{\mathrm{v}} + Q_{\mathrm{n}} + Q_{\mathrm{Cu}} = 0$$

Substituting the heat transfer expressions above into previous equation yields

$$m_{\mathrm{v}} \cdot c_{\mathrm{v}} \cdot \left(J_2 - J_1\right) + C_{\mathrm{n}} \cdot \left(J_2 - J_1\right) + m_{\mathrm{Cu}} \cdot c_{\mathrm{Cu}} \cdot \left(J_2 - J\right) = 0$$

We see that temperatures enter previous equation only as differences. Thus because the intervals on the Celsius and the Kelvin scales are identical, we can use either of these scales in this equation. Solving this equation for J_2 and substituting, we have

$$J_2 = \frac{m_{\mathrm{Cu}} \cdot c_{\mathrm{Cu}} \cdot J + C_{\mathrm{n}} \cdot J_1 + m_{\mathrm{v}} \cdot c_{\mathrm{v}} \cdot J_1}{m_{\mathrm{Cu}} \cdot c_{\mathrm{Cu}} + C_{\mathrm{n}} + m_{\mathrm{v}} \cdot c_{\mathrm{v}}}.$$

The numerator is

$$\left(75\ \mathrm{g}\right) \cdot \left(0{,}092\ \mathrm{cal}/\left(\mathrm{g \cdot K}\right)\right) \cdot \left(312\ °\mathrm{C}\right) + \left(45\ \mathrm{cal/K}\right) \cdot \left(12\ °\mathrm{C}\right) + \left(220\ \mathrm{g}\right) \cdot \left(1\ \mathrm{cal}/\left(\mathrm{g \cdot K}\right)\right) \cdot \left(12\ °\mathrm{C}\right) = 5332{,}8\ \mathrm{cal}$$

and the denominator is

$$\left(75\ \mathrm{g}\right) \cdot \left(0{,}092\ \mathrm{cal}/\left(\mathrm{g \cdot K}\right)\right) + 45\ \mathrm{cal/K} + \left(220\ \mathrm{g}\right) \cdot \left(1\ \mathrm{cal}/\left(\mathrm{g \cdot K}\right)\right) = 271{,}9\ \mathrm{cal/°C}.$$

So, we then have:

$$J_2 = \frac{5332{,}8\ \mathrm{cal}}{271{,}9\ \mathrm{cal/°C}} = 19{,}6\ °\mathrm{C} \cong 20\ °\mathrm{C}$$

From the given data you can show that:

$$Q_{\mathrm{v}} \cong 1670\ \mathrm{cal}, \qquad Q_{\mathrm{n}} \cong 342\ \mathrm{cal}, \qquad Q_{\mathrm{Cu}} \cong -2020\ \mathrm{cal}.$$

Apart from calculation errors due to rounding, the algebraic sum of these three heat transfers is indeed zero, as $Q_{\mathrm{v}} + Q_{\mathrm{n}} + Q_{\mathrm{Cu}} = 0$.

Exercise 13

A tank of water has been outdoors in cold weather and a 5,0-cm-thick slab of ice has formed on its surface. The air above the ice is at –10 °C. Calculate the rate of formation of ice (in centimeters per hour) on the bottom surface of the ice slab. Take the thermal conductivity and density of ice to be 0,004 cal/(s.cm.°C) and 0,92 g/cm^3. Assume that heat is not transferred through the walls or bottom of the tank.