## Face-antimagic labelings

A labeling of type $(1,1,1)$ assigns labels from the set $\{1,2,3, \ldots,|V(G)|+$ $|E(G)|+|F(G)|\}$ to the vertices, edges and faces of plane graph $G$ in such a way that each vertex, edge and face receives exactly one label and each number is used exactly once as a label.
A labeling of type $(1,1,0)$ is a bijection from the set $\{1,2,3, \ldots,|V(G)|+|E(G)|\}$ to the vertices and edges of plane graph $G$.
If we label only vertices (respectively edges, faces) we call such a labeling a vertex (respectively edge, face) labeling and also the labeling is said to be of type ( $1,0,0$ ) (respectively type $(0,1,0))$, type $(0,0,1))$ ).

The weight of a face under a labeling is the sum of labels (if present) carried by that face and the edges and vertices surrounding it.
A labeling of plane graph $G$ is called $d$-antimagic if for every number $s$ the set of $s$-sided face weights is $W_{s}=\left\{a_{s}, a_{s}+d, a_{s}+2 d, \ldots, a_{s}+\left(f_{s}-1\right) d\right\}$ for some integers $a_{s}$ and $d, d \geq 0$, where $f_{s}$ is the number of $s$-sided faces.

We allow different sets $W_{s}$ for different $s$. If $s$ is the same for each face, then there is just one arithmetic sequence comprising the set of face weights and we may speak of a graph being ( $a, d$ )-face antimagic. Many common types of plane graphs have "almost" all faces the same, for example, the prism which consists of all-but-two 4 -sided faces; or the antiprism which consists of all-but-two 3 -sided faces. Such graphs are easily modified so that they contain all the same faces and so that we can consider ( $a, d$ )-face antimagic labeling on them.

If $d=0$ then Ko-Wei Lih called such labeling magic (face-magic).
If $d=1$ then $d$-antimagic labeling is called consecutive.
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