Normalized embedding of trees

The path P_n is the tree with $V(P_n) = \{x_i : 0 \leq i \leq n-1\}$ and $E(P_n) = \{x_i x_{i+1} : 0 \leq i \leq n-2\}$.
We embed the path P_n as a subgraph of the 2-dimensional grid. Given such an embedding, we consider the ordered set of subpaths $L_1, L_2, \ldots, L_k$ which are maximal straight segments in the embedding, where the end of L_i is the beginning of L_{i+1} for any $i = 1, 2, \ldots, k-1$.
Suppose that $L_i \cong P_2$ for some i, $1 < i < k$, $V(L_i) = \{u_0, v_0\}$, thus $u_0 \in V(L_{i-1}) \cap V(L_i)$ and $v_0 \in V(L_i) \cap V(L_{i+1})$. Let $u \in V(L_{i-1}) - \{u_o\}$ and $v \in V(L_{i+1}) - \{v_o\}$ such that their distance on the grid is 1. The replacement of the edge $u_0 v_0$ by the new edge uv is called an *elementary transformation* of the path P_n.

We say that a tree T of order n is a *path-like tree* when it can be obtained after a sequence of elementary transformations on an embedding of P_n in the 2-dimensional grid.
The concept of path-like tree was introduced by C. Barrientos in 2004.

Let $\mathbb{L}$ be the 2-dimensional grid. If we fix a crossing point as $(0,0)$, then each crossing point in $\mathbb{L}$ is perfectly determined by an ordered pair (i,j) where i denotes the row (level) and j denotes the column of $\mathbb{L}$.

Let $\mathbf{I}$ be an embedding of the path P in $\mathbb{L}$ such that:

1. one end vertex of the path P is $(0,0)$,

2. each row of the embedding contains at least two vertices of the path P, and each vertical subpath in the embedding is isomorphic to P_2,

3. assume that i is an even integer and that $(i,j), (i,j+1), (i,j+2), \ldots, (i,j+t)$ is a maximal straight horizontal subpath (isomorphic to P_{t+1}) in the embedding of the path P in $\mathbb{L}$. If $(i+1,m)$ belongs to the embedding of the path P in $\mathbb{L}$, then $m \leq j+t$,

4. assume that i is an odd integer and that $(i,j), (i,j-1), (i,j-2), \ldots, (i,j-s)$ is a maximal straight horizontal subpath (isomorphic to P_{s+1}) in the embedding of the path P in $\mathbb{L}$. If $(i+1,m)$ belongs to the embedding of the path P in $\mathbb{L}$, then $m \geq j-s$.

Then the embedding $\mathbf{I}$ is called a *normalized embedding* of the path P in the grid $\mathbb{L}$.

We study properties of path-like trees which can be obtained from a set of elementary transformations on a normalized embedding of the path in the 2-dimensional grid. We also provide necessary conditions that allow us to exclude trees with maximum degree at most 4 from being path-like trees. Furthermore, we established a relation among the number of normalized embeddings of the path P_n in the 2-dimensional grid, and the Fibonacci numbers.

• Bača, M.- Lin, Y.- Muntaner-Batle, F.A.: *Normalized embedding of path-like trees*, **Utilitas Math. 78** (2009), 11-31.