Antimagic labelings

In 1990, Hartsfield and Ringel introduced the concept of an *antimagic labeling* of a graph. They conjectured that every connected graph other than K_2 is antimagic. An *antimagic labeling* of a graph $G = (V, E)$ is a bijection from E to the a set of positive integer $\{1, 2, \ldots, |E|\}$ such that all vertex weights are pairwise distinct, where a *vertex weight* of a vertex v, $wt(v)$, is defined as the sum of the labels of all the edges incident with the vertex v. A graph G is called *antimagic* if there exists an antimagic labeling of G.

Some families of graphs, for example, path P_m, star S_m, cycle C_m, complete graph K_m, wheel W_m and bipartite graph $K_{2,m}$, $m \geq 3$, have been proved to be antimagic in Hartsfield and Ringel's paper (1990). For more than two decades there have been many attempts to settle the conjecture but although there are now many supporting results, there still are many gaps that need to be filled.

Alon, Kaplan, Lev, Roditty and Yuster in (2004) used several probabilistic tools and some techniques from analytic number theory to show that this conjecture is true for all graphs having minimum degree $\Omega(\log |V(G)|)$. They also proved that if G is a graph with $|V(G)| \geq 4$ vertices and maximum degree $\Delta(G) \geq |V(G)|-2$ then G is antimagic. It is still an open problem to decide whether connected graphs with $\Delta(G) \geq n - k$ and $n > n_0(k)$ are antimagic, for any fixed $k \geq 3$. In the final section of their paper, it is shown that all complete partite graphs, except K_2, are antimagic.

It is not difficult to produce many antimagic labelings for most graphs. We have an interesting situation: On one hand, the Hartsfield and Ringel conjecture remains open in general, despite the efforts of many researchers. On the other hand, whenever an antimagic labeling exists, it seems to be quite easy to find such a labeling. Moreover, often we can find many different antimagic labelings for a given graph.

In the next papers we construct antimagic labelings for regular multipartite graphs and then extend them to other regular graphs.

• Bača, M. Miller, M. - Phanalasy, O.- Semaničová -Feňovčiková, A.: *Constructions of antimagic labelings for some families of regular graphs*, **Journal of Algorithms and Computation 44** (2013), 1-7.